Numerical approximation of some inverse problems

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Abstract In the context of a parameter identification problem, often referred to as an Inverse Problem, the objective is to determine the optimal values of parameters for a model in order to minimize the gap between the predictions of the mathematical model and the empirical or actual observed data.
My presentation focuses on two application domains of the inverse problem:

- The first domain concerns an application in Finance, where we study the identification of derivative terms of financial assets in the context of Partial Differential-Integral Equations (PDIEs) [2]. These equations arise when analyzing option pricing models based on Lévy-type processes (1) and using the Dupire equation.

$$\begin{cases} dX_t &= F(t, X_t, Y_t)dt + \sigma_t X_t dW_t + \displaystyle\int_{\mathbb{R}^n} \gamma(t, X(t^-), z)\widetilde{N}^X(dt, dz), \\ \sigma_t &= f(Y_t), \\ dY_t &= a(t, Y_t)dt + b(t, Y_t)dW'_t, \\ X_0 &= x_0 \in \mathbb{R}^n, \\ Y_0 &= y_0 \in \mathbb{R}^d. \end{cases} \tag{1}$$

  We approach this inverse problem using the Tikhonov regularization method ([1], [3]), which allows a stable reconstruction of model parameters for diffusion with jumps and stochastic volatility from a limited set of observed options.

- The second domain deals with an application in Environment, specifically the identification of the source term in a saline intrusion problem. This issue is of crucial importance in water resource management. By adopting the clear/diffuse sharp interface approach within an unconfined aquifer as proposed in [4] .

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$$\begin{cases} \phi\partial_t h_1 - div(K(T_f(h-h_1)+T_s(h)\chi_0(h_1))\nabla h_1) - div(KT_s(h)\chi_0(h_1)\nabla h) \\ \qquad -div(\delta\phi\nabla h_1) = -Q_sT_s(h) - Q_fT_f(h-h_1), \\ \phi\partial_t h - div(KT_s(h)\chi_0(h_1)\nabla h) - div(KT_s(h)\chi_0(h_1)\nabla h_1) \\ \qquad -div(\delta\phi\nabla h) = -Q_sT_s(h) \end{cases} \quad . \tag{2}$$

The study aims to determine the unknown source term, which is spatially dependent, using boundary data. The mathematical formulation is based on the method of separation of variables [5] . To address the ill-posed nature of the problem, the Tikhonov regularization method is implemented. The numerical results are validated within scenarios involving the pumping and injection of freshwater [6].

Keywords: Inverse problem, Tikhonov regularisation, option pricing, Lévy-type processes, unconfined aquifer, separation of variables.

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