

Analysis of finite volume schemes for the quasi-geostrophic flows at low Froude number

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Abstract

The shallow water equation system, which is derived from depth-averaged Navier-Stokes equations under Boussinesq and hydrostatic pressure assumption, plays an important role in the numerical simulation of oceanic models. This simplified model is widely used to model many phenomena like coastal flows and dam-break floods. There are several kinds of source terms which can be taken into account in this model. For instance, the influence of bottom topography, Manning friction effects and Coriolis force. For large scales oceanic phenomena, the Coriolis force due to the rotation of the Earth plays a central role since the atmospheric or oceanic circulations are frequently observed around the so-called geostrophic equilibrium which corresponds to the balance between the pressure gradient and the Coriolis source term. The ability of numerical schemes to well capture the trivial steady state, i.e. the lake at rest, has been studied in many research articles. However, the question of the geostrophic equilibrium, including the resulting divergence free constraint on the velocity, is much more complex and only few works have been devoted to the preservation of this non trivial equilibrium. It is still an interesting open question and starts to receive a great attention.

In this work, we are interested in designing finite volume schemes with a focus on the preservation of the discrete geostrophic equilibrium to improve significantly the accuracy of numerical simulations when perturbations around this equilibrium are considered. Particularly, we develop collocated and staggered schemes on both rectangular and triangular meshes for the linear wave equation with Coriolis force, which is the linearised model of the original shallow water equations. The crucial common point of the various methods we present is to adapt and combine several strategies known as the Apparent Topography, the Low Mach and the Divergence Penalisation methods, in order to handle correctly the numerical diffusions involved in the schemes on different cell geometries, so that they do not destroy geostrophic equilibria. Finally, we extend these strategies to the non-linear case and show some convincing numerical results.

Keywords: Geostrophic equilibrium, low Froude number, hyperbolic system, finite volume method, Godunov scheme, numerical diffusion, well-balanced scheme, Coriolis force.