ADER method on chimera meshes: application to incompressible Navier-Stokes equations and hyper-reduction for advection-diffusion problems

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We propose a space-time Finite Element (FEM) / Finite Volume (FVM) scheme on moving Chimera grids for a general linear and nonlinear advection-diffusion problem. Special care is devoted to grid overlapping zones in order to devise a compact and accurate discretization stencil to exchange information between different mesh patches. Like in the ADER method, the equations are discretized on a space-time slab. Thus, instead of time-dependent spatial transmission conditions between relatively moving blocks, we define interpolation polynomials on arbitrarily intersecting space-time cells at the block boundaries. Through this scheme, a mesh-free FEM-predictor/FVM-corrector approach is employed for representing the solution. In this discretization framework, a new space-time Local Lax-Friedrichs (LLF) stabilization speed is defined by considering both the advective and diffusive nature of the equation. The numerical illustrations for linear and nonlinear systems show that background and foreground moving meshes do not introduce spurious perturbations to the solution, uniformly reaching second order of accuracy in space and time. It is shown that several foreground meshes, possibly overlapping and with independent displacements, can be employed thanks to this approach.

The main application of the scheme is for the Navier-Stokes equations in order to simulate incompressible viscous flows in an evolving domain. In this case, the employed evolving space-time overset grids are able to take into account both possibly moving objects and the evolution of the domain. Since a classical fractional step method is adopted, a Poisson problem for the pressure needs to be numerically solved. For this reason, for the discretization of the gradient operator, a hybrid technique is defined which is able to automatically encode the particular local overlapping configuration at the interfaces of two blocks. This avoids a subsequent interpolation step at the interface to exchange information between different blocks. The resulting method is second order accurate for both velocity and pressure in space and time. The accuracy and efficiency of the method are tested through reference simulations.

Finally, a reduced and hyper-reduced ADER scheme for general advection-diffusion equations on overset grids is presented. This scheme, based on the Proper Orthogonal Decomposition (POD) approach, allows to reduce the computational costs for both finding the numerical solution and for computing the integrals involved in the definition of the matrices of the algebraic counterpart. In an offline training stage it is built a proper reduced subspace onto which the reduced solution is later projected. Successively, in the online step, a numerical reduced solution is found with respect to a parameter defining the evolution of the domain. In order to alleviate the computational costs for performing the numerical integrals via quadrature rule, a further offline training stage is computed. It allows to define a largely small set of quadrature nodes for any admissible movement of the mesh. The approach is in a Domain Decomposition (DD) frame: consequently over the background mesh the reduced solution is recovered while in background the solution is high-fidelity. The performance of the proposed scheme is tested on both linear and nonlinear problem for different movement of the computational domain. Results show that the computational costs reduce to $O(1)$ degrees of freedom by preserving the accuracy of the solution.