On a Problem Related to Isolated Singularities of Polyharmonic Operator
Abstract

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We consider the equation (in the sense of distribution):

$$\begin{cases} (-\Delta)^m u = g(x,u) \geq 0 \text{ in } \mathcal{D}'(\Omega') \\ (-\Delta)^k u \geq 0 \text{ in } \Omega', \forall k = 0, \cdots m-1, \end{cases}$$

where $\Omega' = \Omega \setminus \{0\}$ and $\Omega \subset \mathbb{R}^{2m}$ is a bounded domain. Then it is known (thanks to Brezis and Lions [1] and Lions [4]) that u solves

$$(-\Delta)^m u = g(x,u) + \sum_{i=0}^{m-1} \alpha_i (-\Delta)^i \delta_0, \text{ in } \mathcal{D}'(\Omega), \tag{0.1}$$

for some nonnegative constants α_i's. In this talk (based on Dhanya R and A.S [3]), we will discuss the existence of singular solutions to

$$(-\Delta)^m u = a(x) f(u) + \sum_{i=0}^{m-1} \alpha_i (-\Delta)^i \delta_0 \text{ in } \mathcal{D}'(\Omega),$$

and a is a nonnegative measurable function in some suitable Lebesgue space. If $(-\Delta)^m u = a(x)f(u)$ in Ω', then we find the growth of the nonlinearity f that determines $\alpha_i = 0$ for all $i = 0, \cdots, m-1$. In case when $\alpha_i = 0$, for all $i = 0, \cdots, m-1$, we will establish regularity results when $f(t) \leq Ce^{\gamma t}$, for some $C,\ \gamma > 0$. This result extends the works of Soranzo [5] and Caristi et al. [2], where the authors find the barrier function in higher dimensions ($N \geq 2m$) with a specific weight function $a(x) = |x|^\sigma$ for $\sigma \in (-2m, 0)$.

References

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